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Thermoelastic response of a heated thin composite plate using the hyperbolic heat conduction model: lumped analysis

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Abstract

The transient thermoelastic response of a thin composite plate composed of a dominant matrix and an insert is investigated. The plate is heated by exposing the matrix to a heating source in the form of a step function. The hyperbolic heat conduction model is used to determine the thermal behavior of the plate, which is assumed to be lumped in the transverse direction. The dominant temperature of the matrix is used to evaluate the thermal stresses. The effects of the heating source intensity and time duration, the insert volume fraction and the convection coefficients on the temperatures and thermal stresses are investigated and presented. The thermal stresses generated within the plate are found to be compressive, follow the behavior of the temperature, and largely affected by the heating source intensity and duration.

Keywords: Thermal behavior; Stresses; Composite plate; Hyperbolic model; Lumped

1. Introduction

Composite structures operate in a variety of thermal environments and can be exposed to severe thermal loading. This might have a pronounced impact on the performance and strength of such structures. The response of composite structures to thermal loading is related to the differences in thermal and mechanical properties of the different constituents, in addition to the nature of the heating process itself.

For situations involving energy sources such as laser and microwave with extremely short duration or very high frequency and very high temperature gradients, heat is found to propagate at a finite speed. To account for the phenomena involving the finite propagation velocity of the thermal wave, the classical Fourier heat flux model should be modified. Catteneo [1] and Vernotte [2] suggested independently a modified heat flux model that takes into account the phase lag in the heat flux.

In the literature, most of the works that investigated the subject of thermal stresses in thin plates assumed the validity of the parabolic diffusion heat conduction model in order to describe the thermal behavior of the plate such as Boley and Weiner [3] and Nowacki [4]. A major qualitative and quantitative change in the behavior of thermal stresses will appear as a result of taking into account the lagging behavior using the hyperbolic and the dual-phase-lag heat conduction model. Tzou [5] addressed the subject of lagging behavior of heat transport in both rigid and deformable bodies. Suh and Burger [6] and Al-Huniti and Al-Nimr [7] assumed the validity of the hyperbolic heat conduction model to investigate the thermal stresses within thin plates exposed to a fast heating rate. Al-Nimr and Al-Huniti [8] investigated the transient thermal stresses generated within a thin homogeneous plate as a result of a fast rate of heating rate using dual-phase-lag heat conduction model.

The present work aims to investigate the transient thermoelastic response of a thin composite plate. The hyperbolic heat conduction model is used to determine the thermal behavior of the plate. The material of the plate is a composite consisting of two domains; a dominant matrix (domain 1) and an insert (domain 2). The thermal behavior is assumed to be lumped in the transverse direction. The effects of different parameters such as the heating source intensity, duration, insert volume fraction, and convection coefficients on the thermoelastic behavior are investigated

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Nomenclature

a	plate side length
Bi	Biot number
c	specific heat
D	plate bending rigidity
E, F	dimensionless convection coefficients
u	unit step function
$\hat{u}$	heat source (per unit volume)
U_o	dimensionless heat source
U_o^*	amplitude of the dimensionless heat source
h	volumetric convection heat transfer coefficient
k	thermal conductivity
$2L$	plate thickness
M_T	thermal moment
$\vec{q}$	heat flux vector
s	Laplace variable
S	dimensionless stress
T	temperature
T_∞	plate initial temperature
t	time
V	volume
w	transverse deflection of the plate
W	Laplace transformation of the dimensionless temperature

Greek symbols

α	coefficient of thermal expansion
ε	volume fraction of the insert (material 2)
η	dimensionless time
η_o	dimensionless pulse duration
γ	thermal diffusivity
μ	Young's modulus
ν	Poisson's ratio
ρ	mass density
σ_x, σ_y	normal stress in the x and y direction, respectively
$\bar{\tau}$	relaxation time
θ	dimensionless temperature
ζ	dimensionless transverse coordinate

2. Formulation of the heat equations

The plate under consideration is shown in Fig. 1. The material of the plate consists of two different materials, a matrix (domain 1) and an insert (domain 2), as shown in Fig. 2. The governing heat equation in each domain is given by

$$(1-\varepsilon)\rho_1 c_1 \frac{\partial T_1}{\partial t} = -(1-\varepsilon)\vec{\nabla}\cdot\vec{q}_1 + (1-\varepsilon)\hat{u}_1 + h(T_2 - T_1) \tag{1}$$

$$\varepsilon\rho_2 c_2 \frac{\partial T_2}{\partial t} = -\varepsilon\vec{\nabla}\cdot\vec{q}_2 + \varepsilon\hat{u}_2 + h(T_1 - T_2) \tag{2}$$

where ε is the insert (material 2) volume fraction, given by $\varepsilon = V_1/(V_1 + V_2)$, ρ is the mass density, c is the specific heat, h is the volumetric convection heat transfer coefficient, $\vec{q}$ is the heat flux vector, T is the temperature, t is the time variable, $\hat{u}$ is the heat source (per unit volume) and the subscripts 1 and 2 refer to domain 1 (matrix) and domain 2 (insert), respectively. Applying the hyperbolic heat conduction model to each domain results in

$$\vec{q}_1 + \bar{\tau}_1 \frac{\partial \vec{q}_1}{\partial t} = -k_1 \vec{\nabla} T_1 \tag{3}$$

$$\vec{q}_2 + \bar{\tau}_2 \frac{\partial \vec{q}_2}{\partial t} = -k_2 \vec{\nabla} T_2 \tag{4}$$

where $\bar{\tau}$ is the relaxation time (phase-lag in the heat flux) and k is the thermal conductivity. Eliminating $\vec{q}_1$ and $\vec{q}_2$ between Eqs. (1), (2) and (3), (4), considering heating only in domain 1 ($\hat{u}_2 = 0$) and assuming lumped behavior in the z-direction yields the governing equations

$$\bar{\tau}_1\rho_1 c_1 \frac{\partial^2 T_1}{\partial t^2} + \rho_1 c_1 \frac{\partial T_1}{\partial t} + \frac{\bar{\tau}_1 h}{1-\varepsilon}\left(\frac{\partial T_1}{\partial t} - \frac{\partial T_2}{\partial t}\right) = \hat{u}_1 + \frac{h}{1-\varepsilon}(T_2 - T_1) + \bar{\tau}_1 \frac{\partial u_{o1}}{\partial t} \tag{5}$$

$$\bar{\tau}_2\rho_2 c_2 \frac{\partial^2 T_2}{\partial t^2} + \rho_2 c_2 \frac{\partial T_2}{\partial t} + \frac{\bar{\tau}_2 h}{\varepsilon}\left(\frac{\partial T_2}{\partial t} - \frac{\partial T_1}{\partial t}\right) = \frac{h}{\varepsilon}(T_1 - T_2) \tag{6}$$

It is worth mentioning here that the lumped behavior in the transverse direction is adopted because the plate is very thin, the heating source under consideration is uniform spatially, and the plate is insulated from both sides. This implies that

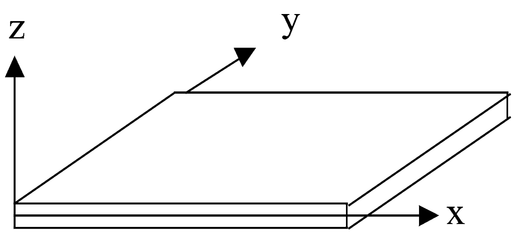

Fig. 1. The geometry of the plate under consideration.

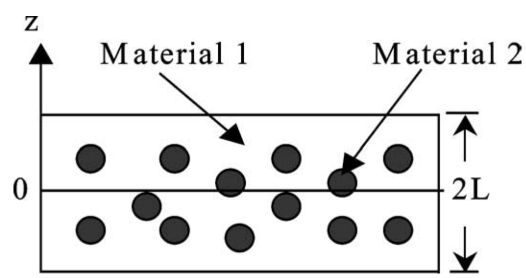

Fig. 2. A view of a section of the plate showing the material composition.

Bi (Biot number) is zero and therefore, thermally lumped behavior is assumed.

Introducing the following dimensionless parameters:

$$\theta = \frac{T - T_\infty}{T_\infty}, \qquad \eta = \frac{\gamma_1 t}{L^2}, \qquad \tau = \frac{\gamma_1 \bar{\tau}}{L^2}$$

where T_∞ is the ambient temperature (which is the same as the initial temperature of the plate), γ is the thermal diffusivity and L is half-thickness of the plate (Fig. 2). Using the dimensionless parameters, Eqs. (5), (6) become

$$\tau_1 \frac{\partial^2 \theta_1}{\partial \eta^2} + \frac{\partial \theta_1}{\partial \eta} + \tau_1 E\left(\frac{\partial \theta_1}{\partial \eta} - \frac{\partial \theta_2}{\partial \eta}\right) = U_o + E(\theta_2 - \theta_1) + \tau_1 \frac{\partial U_o}{\partial \eta} \tag{7}$$

$$\tau_2 \frac{\partial^2 \theta_2}{\partial \eta^2} + \frac{\partial \theta_2}{\partial \eta} + \tau_2 F\left(\frac{\partial \theta_2}{\partial \eta} - \frac{\partial \theta_1}{\partial \eta}\right) = F(\theta_1 - \theta_2) \tag{8}$$

Where in the above equations

$$U_o = \frac{\hat{u}_1 L^2}{k_1 T_\infty}, \qquad E = \frac{hL^2}{(1-\varepsilon)k_1}, \quad \text{and} \quad F = \frac{hL^2 \rho_1 c_1}{\varepsilon \rho_2 c_2 k_1}$$

Initial conditions:

$$\theta_1(0) = \theta_2(0) = 0 \tag{9}$$

Using Laplace transformation technique with the notation $\ell\{\}$ denoting the Laplace transform of $\{\}$, we define the following

$$W_1(s) = \ell\{\theta_1(\eta)\}, \qquad W_2(s) = \ell\{\theta_2(\eta)\} \tag{10}$$

Transforming Eqs. (7) and (8) results in

$$\tau_1 s^2 W_1 + sW_1 + \tau_1 Es(W_1 - W_2) = \ell\{U_o\} + E(W_2 - W_1) + \tau_1\big[s\ell\{U_o\} - U_o(\eta = 0)\big] \tag{11}$$

$$\tau_2 s^2 W_2 + sW_2 + \tau_2 Fs(W_2 - W_1) = F(W_1 - W_2) \tag{12}$$

Note that Eq. (12) can be written as

$$W_2 = \beta W_1 \tag{13}$$

where

$$\beta = \frac{F + \tau_2 Fs}{\tau_2 s^2 + s + \tau_2 Fs + F} \tag{14}$$

Substitute Eq. (13) into Eq. (11) results in

$$W_1 = \frac{(1 + \tau_1 s)\ell\{U_o\} - \tau_1 U_o(\eta = 0)}{\tau_1 s^2 + s + \tau_1 Es - \tau_1 Es\beta - E\beta + E} \tag{15}$$

Now considering the case with a step function heat source, as shown in Fig. 3, in the form

$$U_o(\eta) = U_o^*\big[1 - u(\eta - \eta_o)\big] \tag{16}$$

where U_o^* is the amplitude of the heat source, u is the unit step function and η_o is the duration. Based on this,

$$\ell\{U_o\} = \frac{U_o^*}{s}\big(1 - e^{-\eta_o s}\big) \tag{17}$$

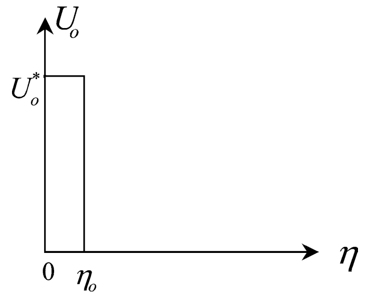

Fig. 3. The dimensionless heat source.

Therefore, Eq. (15) becomes

$$W_1 = \frac{U_o^*[1 - (1 + \tau_1 s)e^{-\eta_o s}]}{s(\tau_1 s^2 + s + \tau_1 Es - \tau_1 Es\beta - E\beta + E)} \tag{18}$$

Solution of the heat equations

The dimensionless temperature variations in the time domain are calculated using the Laplace inversion technique. Equations (18) and (13) are to be inverted and hence, θ_1 and θ_2 determined. A numerical procedure based on the Riemann-sum approximation is used. In this method, any function in the s-domain, $\bar{f}(s, \zeta)$, can be inverted into the time domain $f(\eta, \zeta)$ as:

$$f(\eta, \zeta) = \frac{e^{\varepsilon\eta}}{\eta}\left(\frac{1}{2}\bar{f}(s, \zeta) + Re\sum_{n=1}^{N} \bar{f}\left(\delta + \frac{in\pi}{\eta}, \zeta\right)(-1)^n\right) \tag{19}$$

where, *Re* is the "real part of" and i is the complex number $\sqrt{-1}$. For faster convergence, numerical experiments have shown that the value of δ satisfying the relation ($\delta\eta \approx 4.7$) gives the most satisfactory results [5].

3. Formulation of the thermal stress equations

The plate shown in Fig. 1 is composed of one composite layer. The matrix (domain 1) is being heated by a heat source of the form given in Eq. (16). In the analysis to follow, the temperature of the plate will be taken as that of the matrix, which is the dominant temperature. This is due to the fact that the matrix is the dominant domain ($\varepsilon \leqslant 0.2$). This assumption is also more conservative from a stress point of view since the temperature gradients in the matrix are higher than those in the insert.

In the present analysis it is assumed that the plate is thin and that the classical plate theory applies. Therefore, transverse shear deformations and in-plane deflections are neglected and plane stress conditions apply. It is also assumed that the materials under consideration are homogeneous, isotropic, and linear elastic. Based on this, the normal stresses are given by

$$\sigma_x = \frac{\mu z}{1-\nu^2}\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) - \frac{\mu\alpha}{1-\nu}(T_1 - T_\infty)$$
$$\sigma_y = \frac{\mu z}{1-\nu^2}\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) - \frac{\mu\alpha}{1-\nu}(T_1 - T_\infty) \tag{20}$$

Where w is the transverse deflection of the mid-plane of the plate, μ is Young's modulus, ν is Poisson's ratio and α is the coefficient of thermal expansion.

The equation of motion of the plate as a result of a time-dependent temperature field is given by [10]

$$D\nabla^4 w + \rho(2L)\frac{\partial^2 w}{\partial t^2} = -\frac{\nabla^2 M_T}{1-\nu} \tag{21}$$

where D is the bending rigidity of the plate, given by $D = \mu(2L)^3/[12(1-\nu^2)]$. M_T is the thermal moment defined as

$$M_T = \alpha\mu\int_{-L}^{L}(T_1 - T_\infty)z\,\mathrm{d}z \tag{22}$$

In the present analysis, we consider a simply-supported square plate of side length a. The boundary conditions for this case are given as [3]

$$w(0,y,t) = w(a,y,t) = 0$$
$$w(x,0,t) = w(x,a,t) = 0$$
$$\frac{\partial^2 w(0,y,t)}{\partial x^2} + \frac{M_T}{D(1-\nu)} = \frac{\partial^2 w(a,y,t)}{\partial x^2} + \frac{M_T}{D(1-\nu)} = 0$$
$$\frac{\partial^2 w(x,0,t)}{\partial y^2} + \frac{M_T}{D(1-\nu)} = \frac{\partial^2 w(x,a,t)}{\partial y^2} + \frac{M_T}{D(1-\nu)} = 0 \tag{23}$$

The initial conditions are

$$w(x,y,0) = \frac{\partial w}{\partial t}(x,y,0) = 0 \tag{24}$$

where it is assumed that the plate was at rest in the reference position just before the application of the heat source.

Since in the present work we are dealing with a lumped behavior in the z-direction, then $(T_1 - T_\infty)$ is independent of z. Therefore, the thermal moment, Eq. (22), becomes zero. Looking back at Eqs. (20)–(24), it can be concluded that the main contribution to the thermal stresses come from the last terms in Eq. (20) which becomes

$$\sigma_x = \sigma_y = -\frac{\mu\alpha}{1-\nu}(T_1 - T_\infty) \tag{25}$$

Defining a dimensionless stress as

$$S = \frac{(1-\nu)}{\mu\alpha T_\infty}\sigma_x = \frac{(1-\nu)}{\mu\alpha T_\infty}\sigma_y$$

Eq. (25) becomes

$$S = -\theta_1 \tag{26}$$

Therefore, the dimensionless thermal stress is compressive and has the same behavior as that of the dimensionless temperature.

Note that in the above equations μ, ν, α and ρ are the "equivalent" properties of the composite material. In the present work, both the matrix (domain 1) and the insert (domain 2) are made each of a homogeneous, isotropic material. Therefore, the equivalent properties used are found as follows, where p refers to the property (μ, ν, α or ρ), [9]

$$p = (1-\varepsilon)p_1 + \varepsilon p_2 \tag{27}$$

4. Results and discussion

The materials used for the two layers are Copper (Cu) for the matrix (domain 1) and Lead (Pb) for the insert (domain 2). The properties of the two materials are given in Table 1.

Figs. 4 and 5 show the transient temperature distribution in both domains of the composite plate at different dimensionless intensities of the heating source. As predicted, there is a sharp increase in the matrix temperature within which the heating source evolves its energy. This sharp increase occurs during the duration of the heating source which remains active from $\eta = 0$ to $\eta_o = 0.1$. After that there is a slight decrease in the matrix temperature because it looses part of its gained energy to the insert which does not contain a generating heating source. The temperature of the matrix attains its maximum value at the end of the heating process when the heating source evolves all of its energy and this occurs at $\eta = \eta_o$ as clear in Fig. 4. In contrary to the sharp increase in the matrix temperature, the temperature of the insert increases gradually until it reaches an asymptotic steady state value as shown in Fig. 5. This is because the insert gains the energy of the heating source indirectly through the matrix and so it takes time to feel the heating process. Also, in contrary to the slight decrease in the matrix temperature due to the energy lost from it, there is a significant increase in the temperature of the insert which gains the same amount of energy. This is attributed to the small volume fraction of the insert ($\varepsilon = 0.1$) which implies that small amount of gained energy is able to cause significant increase in the light domain. It is also clear from Figs. 4 and 5 that the temperatures of both domains are linearly proportional to the intensity of

Table 1
Properties of the two materials used in the calculations

Property	Copper (Cu)	Lead (Pb)
k [$\mathrm{W\cdot m^{-1}\cdot K^{-1}}$]	386	35
C [$\mathrm{J\cdot kg^{-1}\cdot K^{-1}}$]	385	129
$\bar{\tau}$ [sec]	0.4348 (10^{-12})	0.1670 (10^{-12})
γ [$\mathrm{m^2\cdot sec^{-1}}$]	1.1283 (10^{-4})	0.2301 (10^{-4})
α [$\mathrm{1\cdot K^{-1}}$]	1.76 (10^{-5})	2.89 (10^{-5})
μ [$\mathrm{N\cdot m^{-2}}$]	119.0 (10^9)	16.0 (10^9)
ν	0.33	0.44
ρ [$\mathrm{kg\cdot m^{-3}}$]	8960	11360

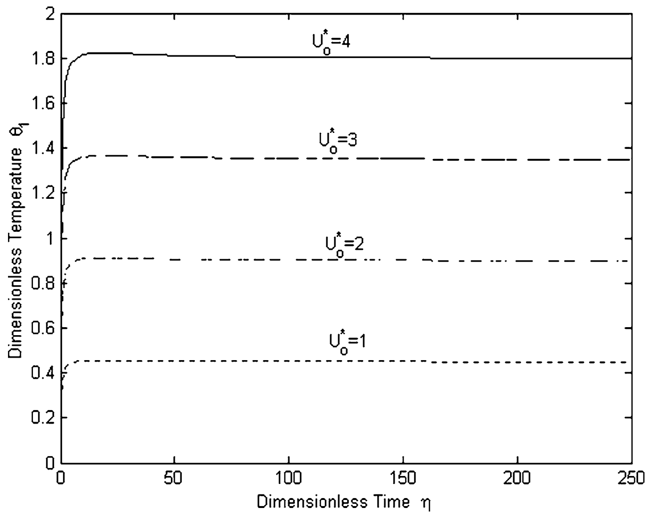

Fig. 4. Variations of the dimensionless temperature in the matrix with dimensionless time for different values of U_o^* ($\varepsilon = 0.1$, $\eta_o = 0.1$, $E = 0.0028785$, $F = 0.06098$).

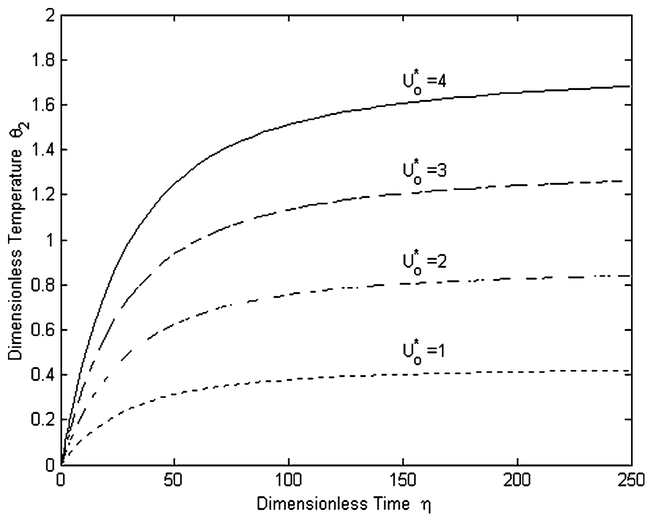

Fig. 5. Variations of the dimensionless temperature in the insert with dimensionless time for different values of U_o^* ($\varepsilon = 0.1$, $\eta_o = 0.1$, $E = 0.0028785$, $F = 0.06098$).

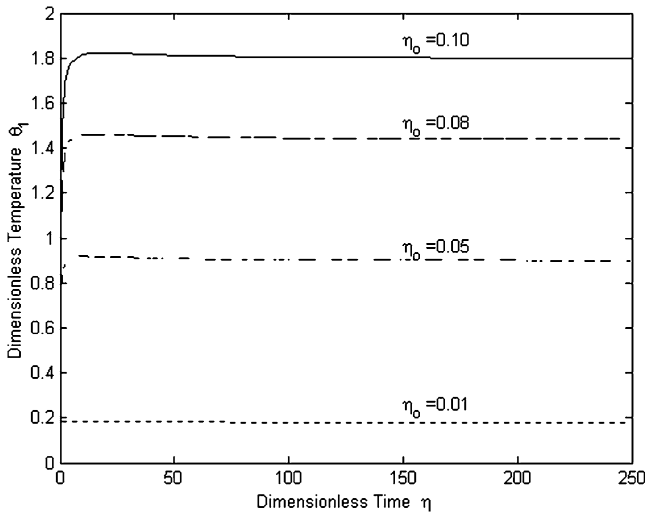

Fig. 6. Variations of the dimensionless temperature in the matrix with dimensionless time for different values of η_o ($\varepsilon = 0.1$, $U_o^* = 4$, $E = 0.0028785$, $F = 0.06098$).

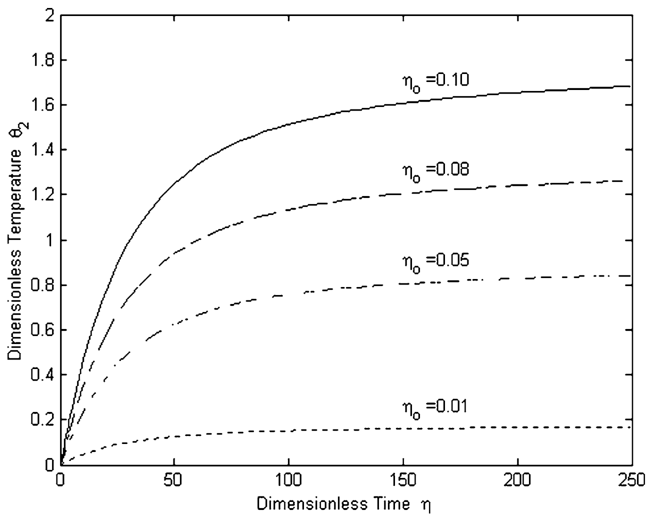

Fig. 7. Variations of the dimensionless temperature in the insert with dimensionless time for different values of η_o ($\varepsilon = 0.1$, $U_o^* = 4$, $E = 0.0028785$, $F = 0.06098$).

the heating source at steady state conditions. This is because the thin slab is perfectly thermally insulated from both sides and all of the evolved energy will be stored as a sensible heat within the slab and will raise the slab temperature in a linear manner.

Figs. 6 and 7 show the effect of the heating source duration on the transient thermal response of both domains. As the duration of the unit-step heating source η_o decreases, the rise in the matrix temperature becomes sharper and the temperature level decreases. This decrease in temperature level is attributed to the decrease in the total amount of energy evolved from the heating source as a result of the decrease in its duration. The same observation is true in the case of the insert as clear from Fig. 7. In the steady state limit, a linear proportionality is observed between the temperature of both domains and the duration of the heating source. Again, this is due to the fact that the total amount of energy evolved within the slab is linearly proportional to η_o and it is given as $\eta_o U_o^*$. The same observation regarding the slight decrease in the matrix temperature and the significant increase in insert temperature appears also in Figs. 6 and 7 and this is discussed earlier.

Figs. 8 and 9 show the effect of the volume fraction on the transient thermal response of both domains. Increasing the volume fraction of the insert causes a significant reduction in the matrix temperature. As mentioned previously, the insert does not contain any heating source and it gains its energy from the matrix. The total thermal capacity of the insert increases as its volume fraction increases and as a result, more energy is lost from the matrix to the insert. On the other

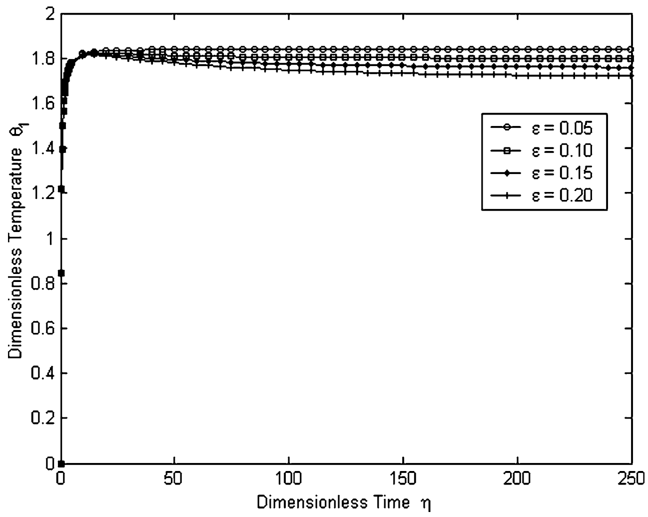

Fig. 8. Variations of the dimensionless temperature in the matrix with dimensionless time for different values of ε ($U_o^* = 4$, $\eta_o = 0.1$, $E = 0.0028785$, $F = 0.06098$).

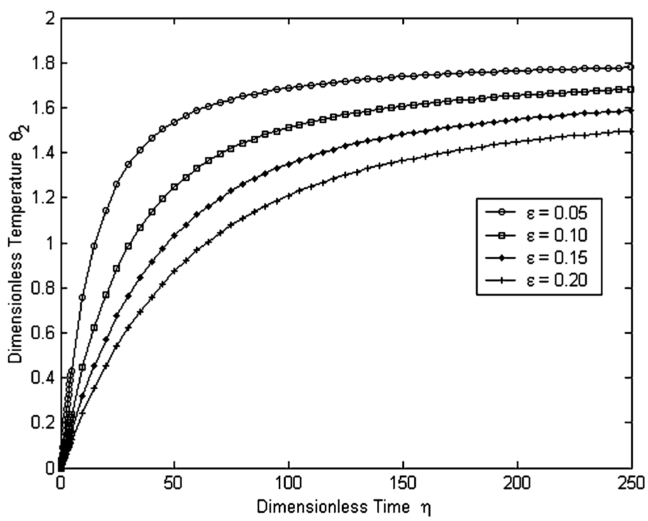

Fig. 9. Variations of the dimensionless temperature in the insert with dimensionless time for different values of ε ($U_o^* = 4$, $\eta_o = 0.1$, $E = 0.0028785$, $F = 0.06098$).

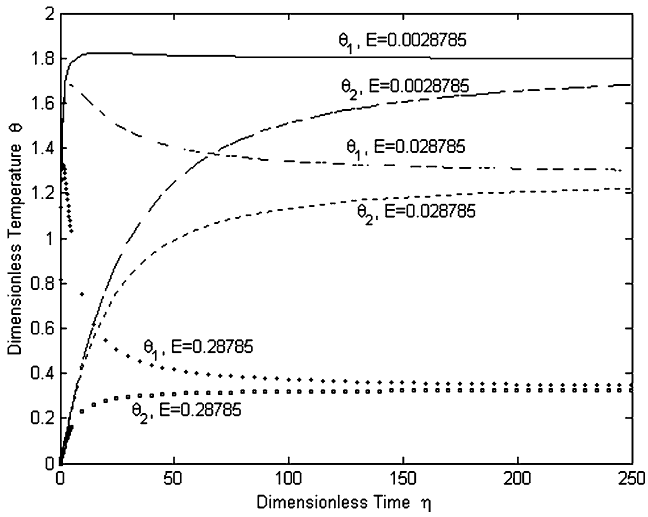

Fig. 10. Variations of the dimensionless temperatures with dimensionless time for different values of E ($\varepsilon = 0.1$, $U_o^* = 4$, $\eta_o = 0.1$, $F = 0.06098$).

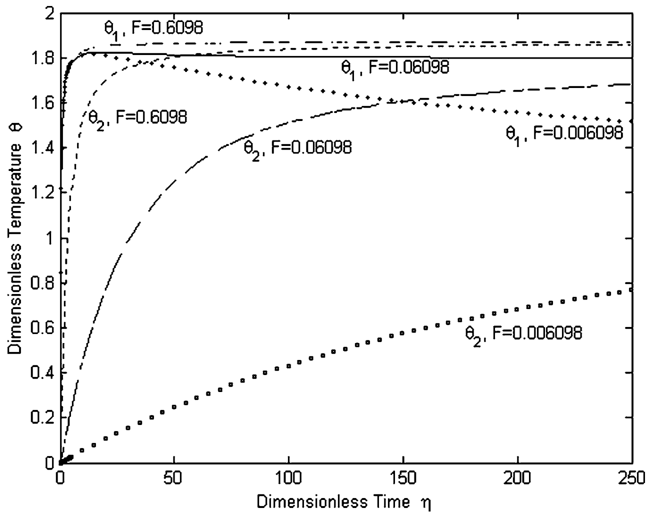

Fig. 11. Variations of the dimensionless temperatures with dimensionless time for different values of F ($\varepsilon = 0.1$, $U_o^* = 4$, $\eta_o = 0.1$, $E = 0.0028785$).

hand, the energy gained by the insert is not able to cause a significant increase in its temperature due to the increase in its total thermal capacity.

Figs. 10 and 11 show the effect of the dimensionless parameters E and F on the transient response of both domains. The parameters E and F represent a sort of dimensionless convective heat transfer coefficient. Increasing these two parameters improves the heat transfer from the matrix to the insert and as a result the deviation between both temperatures decreases. This is obvious from the predictions of the two figures and may be concluded from the governing equations as well.

Figs. 12–14 show the transient variations of the dimensionless thermal stress within the plate with dimensionless time for the same parameters used in the temperature variations and shown previously by Figs. 4–9. Three main factors discussed earlier explain the general behavior in these figures. First, the assumption that the dominant temperature of the plate is taken as that of the matrix. Second, as shown by Eq. (26), the thermal stress in the plate is compressive. Finally, and also as shown by Eq. (26), the (compressive) thermal stress behavior is the same as that of the dimensionless temperature of the matrix. Therefore, Figs. 12–14 represent the negative image of Figs. 4, 6, and 8. Based on this, the previous discussion regarding the temperature of the matrix explains the behavior of the thermal stress in Figs. 12–14.

In Fig. 12, it can be concluded that increasing the intensity of the heating source increases the compressive thermal stress and brings the situation closer to a failure possibility. Same situation arises in Fig. 13 with the increase of the unit step heating source duration. Although the effect of the insert volume fraction is seen to be less pronounced

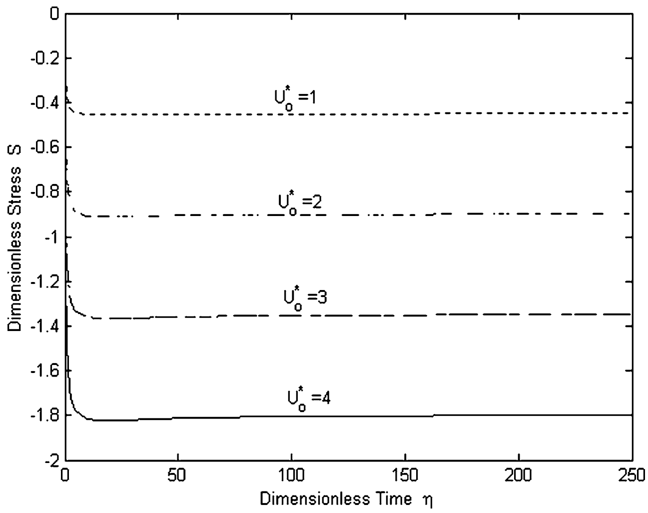

Fig. 12. Variations of the dimensionless stress with dimensionless time for different values of U_O^* ($\varepsilon = 0.1$, $\eta_O = 0.1$, $E = 0.0028785$, $F = 0.06098$).

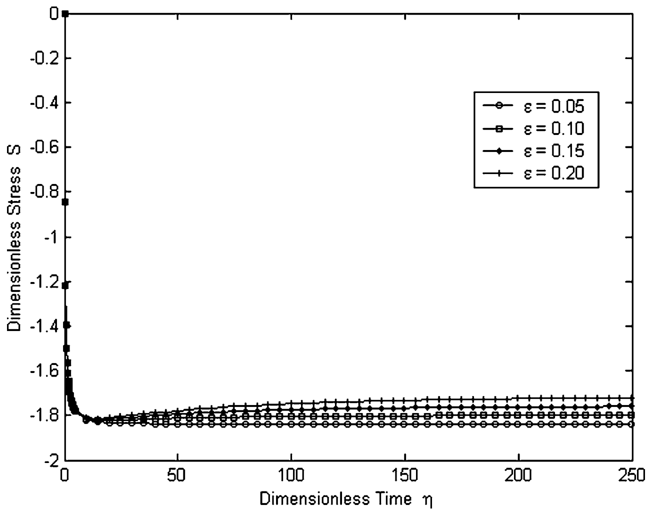

Fig. 14. Variations of the dimensionless stress with dimensionless time for different values of ε ($U_O^* = 4$, $\eta_O = 0.1$, $E = 0.0028785$, $F = 0.06098$).

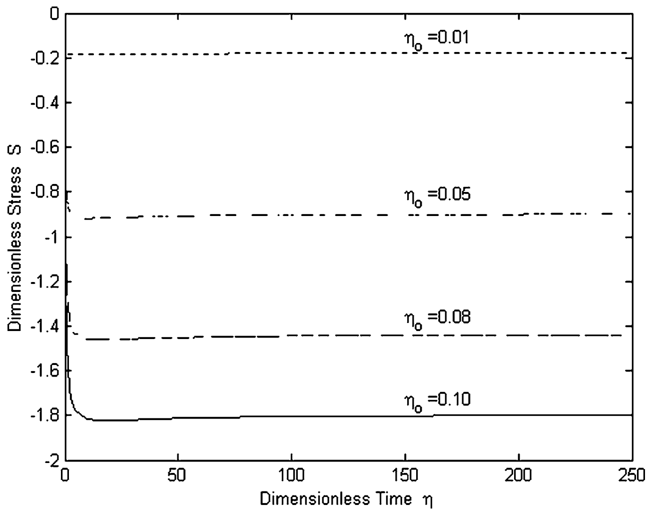

Fig. 13. Variations of the dimensionless stress with dimensionless time for different values of η_O ($\varepsilon = 0.1$, $U_O^* = 4$, $E = 0.0028785$, $F = 0.06098$).

on the thermal stress (Fig. 14), it is clear however that as ε increases, the compressive thermal stress decreases since the volume of the matrix (within which heat is being generated) decreases and so is its effect as well.

5. Concluding remarks

In this work, the transient thermal behavior and the thermal stresses generated within a thin composite plate are determined. The plate is composed of a dominant matrix (domain 1) and an insert (domain 2). Heat is generated inside the matrix by a heating source in the form of step function while the insert has no heat generation. The hyperbolic heat conduction model is used to determine the thermal behavior of the plate in the form of the temperature variations in both domains. The thermal behavior is assumed to be lumped in the transverse direction. In fact, and due to the nature of the heating source, we are considering all the plate to be lumped spatially in all directions. The only variation in temperature is due to transient effects.

The temperature of the matrix is the dominant and this temperature is used to evaluate the thermal stresses generated within the plate. The effects of the heating source intensity and time duration, the insert volume fraction and the convection coefficients on the temperatures and thermal stresses are investigated and presented.

It is found that the resulting thermal stresses are compressive and follow the behavior of the temperature. The effects of increasing the heating source intensity and time duration are found to have a pronounced effect on the thermal stresses.

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